Chemistry Key Equations

Worked Solutions for All Practice Problems

Contents

1 Electrochemistry

1.1 EC1: $E^\circ_{\text{cell}} = E^\circ_{\text{cathode}} - E^\circ_{\text{anode}}$

Simple: Standard cell potential of Cu/Zn cell

Given: $E^\circ(\text{Cu}^{2+}/\text{Cu}) = +0.34$ V (cathode), $E^\circ(\text{Zn}^{2+}/\text{Zn}) = -0.76$ V (anode).

$$\begin{aligned} E^\circ_{\text{cell}} &= E^\circ_{\text{cathode}} - E^\circ_{\text{anode}} \\ &= (+0.34) - (-0.76) \\ &= \boxed{+1.10 \text{ V}} \end{aligned}$$

Unit Conversion: Ag/Fe cell in millivolts

Given: $E^\circ(\text{Ag}^{+}/\text{Ag}) = +0.80$ V, $E^\circ(\text{Fe}^{2+}/\text{Fe}) = -0.44$ V.

$$\begin{aligned} E^\circ_{\text{cell}} &= 0.80 - (-0.44) = 1.24 \text{ V} \\ &= 1.24 \text{ V} \times \frac{1000 \text{ mV}}{1 \text{ V}} = \boxed{1240 \text{ mV}} \end{aligned}$$

Multi-Step: Best pair from three half-cells

Given: $E^\circ(\text{Ni}^{2+}/\text{Ni}) = -0.26$ V, $E^\circ(\text{Sn}^{2+}/\text{Sn}) = -0.14$ V, $E^\circ(\text{Ag}^{+}/\text{Ag}) = +0.80$ V.
Strategy: Maximize E°_{cell} by choosing the most positive cathode and most negative anode.

$$\begin{aligned} \text{Best cathode:} &\quad \text{Ag}^{+}/\text{Ag} = +0.80 \text{ V} \\ \text{Best anode:} &\quad \text{Ni}^{2+}/\text{Ni} = -0.26 \text{ V} \\ E^\circ_{\text{cell}} &= 0.80 - (-0.26) = \boxed{1.06 \text{ V}} \end{aligned}$$

1.2 EC2: $E^\circ_{\text{cell}} = \frac{RT}{nF} \ln K$

Simple: Finding E° from K

Given: $K = 1.0 \times 10^{10}$, $n = 2$, $T = 298$ K.

$$\begin{aligned} E^\circ_{\text{cell}} &= \frac{0.0257}{n} \ln K = \frac{0.0257}{2} \ln(1.0 \times 10^{10}) \\ &= 0.01285 \times 23.03 = \boxed{0.296 \text{ V}} \end{aligned}$$

Unit Conversion: Finding K from E°

Given: $E^\circ = 0.150$ V, $n = 1$.

$$\begin{aligned} \ln K &= \frac{nFE^\circ}{RT} = \frac{(1)(96485)(0.150)}{(8.314)(298)} = 5.84 \\ K &= e^{5.84} = \boxed{339} \end{aligned}$$

Multi-Step: E° and K for Ag/Cu cell

Step 1: Find E°_{cell}:

$$E^\circ_{\text{cell}} = 0.80 - 0.34 = 0.46\ \text{V}$$

Step 2: Find K ($n = 2$ because Cu loses $2e^-$):

$$\begin{aligned} \log K &= \frac{nE^\circ}{0.0592} = \frac{2(0.46)}{0.0592} = 15.54 \\ K &= 10^{15.54} = \boxed{3.9 \times 10^{15}} \end{aligned}$$

1.3 EC3: $E^\circ_{\text{cell}} = \frac{0.0592}{n} \log K$ (at 298 K)

Simple: E° from K using log form

$$E^\circ_{\text{cell}} = \frac{0.0592}{1} \log(1.0 \times 10^6) = 0.0592 \times 6 = \boxed{0.355\ \text{V}}$$

Unit Conversion: Finding K and ΔG° combined

$$\begin{aligned} \log K &= \frac{nE^\circ}{0.0592} = \frac{3(0.50)}{0.0592} = 25.34 \\ K &= 10^{25.34} = 2.2 \times 10^{25} \\ \Delta G^\circ &= -nFE^\circ = -(3)(96485)(0.50) = -144{,}728\ \text{J} = \boxed{-144.7\ \text{kJ/mol}} \end{aligned}$$

Multi-Step: K for Daniell cell and the $K = 1$ condition

Step 1: $E^\circ = 1.10$ V, $n = 2$:

$$\log K = \frac{2(1.10)}{0.0592} = 37.16 \implies K = \boxed{1.5 \times 10^{37}}$$

Step 2: For $K = 1$: $\log(1) = 0$, so $E^\circ_{\text{cell}} = 0$ V. *At equilibrium, there is no net driving force.*

1.4 EC4–5: Nernst Equation

Simple: Nernst with $Q = 1$

$$E = E^\circ - \frac{0.0592}{n} \log Q = 1.10 - \frac{0.0592}{2} \log(1) = 1.10 - 0 = \boxed{1.10\ \text{V}}$$

Unit Conversion: Daniell cell at nonstandard conditions

Given: $E^\circ = 1.10$ V, $n = 2$, $Q = [\text{Zn}^{2+}]/[\text{Cu}^{2+}] = 2.0/0.010 = 200$.

$$\begin{aligned} E &= 1.10 - \frac{0.0592}{2} \log(200) = 1.10 - 0.0296(2.301) \\ &= 1.10 - 0.068 = \boxed{1.032\ \text{V} = 1032\ \text{mV}} \end{aligned}$$

Multi-Step: Concentration cell

Step 1: Both electrodes are Cu, so $E^\circ_{\text{cell}} = 0$.
Step 2: The dilute side is oxidized (anode):

$$Q = \frac{[\text{Cu}^{2+}]_{\text{anode}}}{[\text{Cu}^{2+}]_{\text{cathode}}} = \frac{0.0010}{1.5} = 6.67 \times 10^{-4}$$

Step 3:

$$E = 0 - \frac{0.0592}{2}\log(6.67 \times 10^{-4}) = -0.0296(-3.176) = \boxed{0.094\text{ V}}$$

1.5 EC6–7: $\Delta G = -nFE_{\textbf{cell}}$

Simple: ΔG from cell voltage

$$\Delta G = -(2)(96{,}485)(1.10) = -212{,}267\text{ J} = \boxed{-212.3\text{ kJ}}$$

Unit Conversion: Fuel cell ΔG in kcal

$$\begin{aligned}\Delta G &= -(4)(96{,}485)(0.90) = -347{,}346\text{ J} = -347.3\text{ kJ}\\ &= \frac{-347.3}{4.184} = \boxed{-83.0\text{ kcal/mol}}\end{aligned}$$

Multi-Step: ΔG at nonstandard conditions

Step 1: Nernst:

$$E = 1.10 - \frac{0.0592}{2}\log(100) = 1.10 - 0.0592 = 1.041\text{ V}$$

Step 2:

$$\Delta G = -(2)(96{,}485)(1.041) = \boxed{-200.9\text{ kJ}}$$

1.6 EC8: $w_{\textbf{elec}} = w_{\text{max}} = -nFE_{\textbf{cell}}$

Simple: Maximum work

$$w_{\text{max}} = -(2)(96{,}485)(1.50) = \boxed{-289.5\text{ kJ/mol}}$$

Unit Conversion: Cell voltage from work; conversion to eV

$$E = \frac{|w_{\text{max}}|}{nF} = \frac{400{,}000}{2 \times 96{,}485} = 2.073\text{ V}$$

$$\text{Per electron:}\quad 2.073\text{ V} = \boxed{2.073\text{ eV}}$$

(Since 1 eV = energy gained by $1e^-$ through 1 V potential.)

Multi-Step: Work at nonstandard conditions for partial reaction

Step 1: $E = 1.10 - \frac{0.0592}{2}\log(50) = 1.10 - 0.050 = 1.050$ V
Step 2: $w_{\text{max}}/\text{mol} = -(2)(96485)(1.050) = -202.6$ kJ/mol
Step 3: For 0.50 mol Zn: $w = 0.50 \times (-202.6) = \boxed{-101.3 \text{ kJ}}$

1.7 EC9: $Q = It = nF$ (Electrolysis)

Simple: Total charge

$$Q = It = (5.00)(3600) = \boxed{18{,}000 \text{ C}}$$

Unit Conversion: Moles of electrons from current and time

$$t = 45.0 \text{ min} \times 60 = 2700 \text{ s}$$
$$Q = (2.50)(2700) = 6750 \text{ C}$$
$$n_{e^-} = \frac{6750}{96{,}485} = \boxed{0.0699 \text{ mol } e^-}$$

Multi-Step: Mass of copper deposited

Step 1: Charge: $Q = (3.00)(2.00 \times 3600) = 21{,}600$ C
Step 2: Moles of electrons: $n_{e^-} = 21{,}600/96{,}485 = 0.2239$ mol
Step 3: $Cu^{2+} + 2e^- \rightarrow Cu$, so mol Cu $= 0.2239/2 = 0.1120$
Step 4: Mass: $0.1120 \times 63.55 = \boxed{7.12 \text{ g Cu}}$

2 Thermodynamics

2.1 TD1: $\Delta S = q_{\text{rev}}/T$

Simple: Basic entropy change

$$\Delta S = \frac{5000}{350} = \boxed{14.3 \text{ J/K}}$$

Unit Conversion: Unit conversions required

$$T = 75 + 273.15 = 348.15 \text{ K}; \quad q = 2.50 \text{ kJ} = 2500 \text{ J}$$
$$\Delta S = \frac{2500}{348.15} = \boxed{7.18 \text{ J/K}}$$

Multi-Step: Entropy of melting ice

Step 1: $n = 36.0/18.02 = 2.00$ mol
Step 2: $q = 2.00 \times 6.01 = 12.02$ kJ $= 12{,}020$ J
Step 3: $T = 0°\text{C} = 273.15$ K

$$\Delta S = \frac{12{,}020}{273.15} = \boxed{44.0 \text{ J/K}}$$

2.2 TD4: $\Delta S^\circ = \sum \nu S^\circ(\textbf{products}) - \sum \nu S^\circ(\textbf{reactants})$

Simple: Simple reaction entropy

$$\Delta S^\circ = 200 - 150 = \boxed{50 \text{ J/(mol·K)}}$$

Unit Conversion: $2\text{NO} + \text{O}_2 \rightarrow 2\text{NO}_2$

$$\begin{aligned}\Delta S^\circ &= 2(240.1) - [2(210.8) + 205.2]\\ &= 480.2 - 626.8 = -146.6 \text{ J/(mol·K)} = \boxed{-0.1466 \text{ kJ/(mol·K)}}\end{aligned}$$

Multi-Step: CaCO_3 decomposition: finding the crossover temperature

Step 1: $\Delta S^\circ = (38.1 + 213.8) - 91.7 = 160.2$ J/(mol·K)
Step 2: Set $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ = 0$:

$$T = \frac{\Delta H^\circ}{\Delta S^\circ} = \frac{178{,}300}{160.2} = \boxed{1113 \text{ K} = 840°\text{C}}$$

Above this temperature, the reaction is spontaneous.

2.3 TD8: $\Delta G = \Delta H - T\Delta S$

Simple: Straightforward Gibbs energy

$$\Delta G = -100{,}000 - 298(50) = -100{,}000 - 14{,}900 = \boxed{-114.9 \text{ kJ}}$$

Unit Conversion: Mixed units: kcal to kJ, kJ/K

$$\Delta H = -40.0 \times 4.184 = -167.4 \text{ kJ}; \quad T = 310.15 \text{ K}$$

$$\Delta G = -167.4 - 310.15(0.100) = -167.4 - 31.0 = \boxed{-198.4 \text{ kJ}}$$

Multi-Step: Finding the crossover temperature

Set $\Delta G = 0$: $T_{\text{cross}} = \Delta H/\Delta S = 50{,}000/150 = 333$ K.
Verify at 298 K: $\Delta G = 50 - 298(0.150) = +5.3$ kJ (nonspontaneous ✓)
Verify at 500 K: $\Delta G = 50 - 500(0.150) = \boxed{-25 \text{ kJ}}$ (spontaneous ✓)

2.4 TD9: $q = cm\Delta T$

Simple: Heating water

$$q = (4.184)(100.0)(80.0 - 20.0) = (4.184)(100.0)(60.0) = \boxed{25.1 \text{ kJ}}$$

Unit Conversion: Heating iron (lb to g conversion)

$$m = 2.50 \times 453.6 = 1134 \text{ g}$$
$$q = (0.449)(1134)(175) = \boxed{89.1 \text{ kJ}}$$

Multi-Step: Calorimetry: identify an unknown metal

Step 1: Heat gained by water:

$$q_w = (4.184)(150.0)(24.5 - 22.0) = 1569 \text{ J}$$

Step 2: Heat lost by metal ($q_m = -q_w$):

$$c = \frac{1569}{(50.0)(95.0 - 24.5)} = \frac{1569}{3525} = 0.445 \text{ J/(g·°C)}$$

Step 3: Compare to table: $c \approx 0.449$ J/(g·°C) $\Longrightarrow$ $\boxed{\text{Iron (Fe)}}$

2.5 TD10: $\Delta U = q + w$

Simple: Basic First Law

$$\Delta U = 500 + 200 = \boxed{700 \text{ J}}$$

Unit Conversion: L·atm to J conversion

$$w = -1.20 \text{ L·atm} \times 101.325 \text{ J/(L·atm)} = -121.6 \text{ J}$$
$$\Delta U = 3500 + (-121.6) = \boxed{3378 \text{ J} = 3.38 \text{ kJ}}$$

Multi-Step: Expansion against constant pressure

Step 1: Work:

$$w = -P_{\text{ext}}\Delta V = -(1.00)(10.0 - 5.00) = -5.00 \text{ L·atm} = -506.6 \text{ J}$$

Step 2: $\Delta U = 800 + (-506.6) = +293$ J

Step 3: Since $\Delta U > 0$, the internal energy increased $\Longrightarrow$ $\boxed{\text{temperature increased}}$.

2.6 TD11: $\Delta H^\circ_{\mathbf{rxn}} = \sum n\Delta H^\circ_f(\mathbf{products}) - \sum n\Delta H^\circ_f(\mathbf{reactants})$

Simple: Simple formation enthalpy

$$\Delta H^\circ = (-350) - (-200) = \boxed{-150 \text{ kJ}}$$

Unit Conversion: Combustion of methane

$$\begin{aligned}\Delta H^\circ &= [-393.5 + 2(-285.8)] - [-74.8 + 0]\\ &= -965.1 - (-74.8) = -890.3 \text{ kJ/mol}\\ &= -890.3/4.184 = \boxed{-212.8 \text{ kcal/mol}}\end{aligned}$$

Multi-Step: Combustion of ethane: per gram

Step 1:

$$\begin{aligned}\Delta H^\circ &= [4(-393.5) + 6(-285.8)] - [2(-84.7) + 0]\\ &= [-1574 - 1714.8] - [-169.4] = -3119.4 \text{ kJ}\end{aligned}$$

Step 2: Per gram of C_2H_6 ($M = 30.07$):

$$\frac{-3119.4}{2 \times 30.07} = \boxed{-51.9 \text{ kJ/g}}$$

3 Equilibrium

3.1 EQ1: $Q_c = \frac{[\mathbf{C}]^x[\mathbf{D}]^y}{[\mathbf{A}]^m[\mathbf{B}]^n}$

Simple: Reaction quotient for Haber process

$$Q_c = \frac{[\text{NH}_3]^2}{[\text{N}_2][\text{H}_2]^3} = \frac{(0.20)^2}{(0.50)(0.30)^3} = \frac{0.040}{0.0135} = \boxed{2.96}$$

Unit Conversion: mmol/L to M conversion

500 mmol/L = 0.500 M, etc. Identical calculation: $Q_c = \boxed{2.96}$.

Multi-Step: ICE table for A $\rightleftharpoons$ 2B

Step 1: $Q_{\text{initial}} = 0^2/1.0 = 0 < K = 4.0 \implies$ shifts right.
Step 2: ICE table: $[\text{A}] = 1.0 - x$, $[\text{B}] = 2x$.

$$\frac{(2x)^2}{1.0 - x} = 4.0 \implies 4x^2 + 4x - 4 = 0 \implies x = 0.618$$

$$\boxed{[\text{A}] = 0.382 \text{ M}, \quad [\text{B}] = 1.236 \text{ M}}$$

3.2 EQ6: $K_p = K_c(RT)^{\Delta n}$

Simple: K_p from K_c for Haber process

$$\Delta n = 2 - 4 = -2$$

$$K_p = 0.500 \times (0.08206 \times 500)^{-2} = \frac{0.500}{(41.03)^2} = \boxed{2.97 \times 10^{-4}}$$

Unit Conversion: K_c from K_p with temperature conversion

$T = 227 + 273 = 500$ K; $\Delta n = -1$:

$$K_c = \frac{K_p}{(RT)^{-1}} = K_p \times RT = 1.8 \times 10^{-4} \times 41.03 = \boxed{7.39 \times 10^{-3}}$$

Multi-Step: K_c and equal concentrations

$\Delta n = 3 - 2 = 1$:

$$K_c = \frac{K_p}{RT} = \frac{1.60}{0.08206 \times 1000} = 0.0195$$

If $[SO_3] = [SO_2] = [O_2] = x$:

$$\frac{x^2 \cdot x}{x^2} = x = 0.0195 \implies \boxed{x = 0.0195\ \text{M}}$$

3.3 EQ7–8: Solubility Product (K_{sp})

Simple: K_{sp} of $BaSO_4$

$BaSO_4 \rightleftharpoons Ba^{2+} + SO_4^{2-}$; both $= s = 1.05 \times 10^{-5}$ M.

$$K_{sp} = s^2 = (1.05 \times 10^{-5})^2 = \boxed{1.10 \times 10^{-10}}$$

Unit Conversion: Solubility of AgCl in mg/L to K_{sp}

$$s = \frac{1.9 \times 10^{-3}\ \text{g/L}}{143.32\ \text{g/mol}} = 1.33 \times 10^{-5}\ \text{M}$$

$$K_{sp} = s^2 = \boxed{1.76 \times 10^{-10}}$$

Multi-Step: Will a precipitate form? (mixing problem)

After mixing (volumes double, concentrations halve):

$$[\mathrm{Ag}^+] = \frac{0.0010 \times 50.0}{100.0} = 5.0 \times 10^{-4}\ \mathrm{M}$$
$$[\mathrm{Cl}^-] = \frac{0.0020 \times 50.0}{100.0} = 1.0 \times 10^{-3}\ \mathrm{M}$$
$$Q = (5.0 \times 10^{-4})(1.0 \times 10^{-3}) = 5.0 \times 10^{-7}$$

$Q = 5.0 \times 10^{-7} > K_{sp} = 1.8 \times 10^{-10} \implies \boxed{\text{Precipitate forms!}}$

4 Acid–Base Chemistry

4.1 AB1: $K_w = [\mathbf{H_3O^+}][\mathbf{OH^-}] = 1.0 \times 10^{-14}$

Simple: Finding $[\mathrm{OH}^-]$

$$[\mathrm{OH}^-] = \frac{1.0 \times 10^{-14}}{1.0 \times 10^{-3}} = \boxed{1.0 \times 10^{-11}\ \mathrm{M}}$$

Unit Conversion: mmol/mL to M conversion

$0.025\ \mathrm{mmol/mL} = 0.025\ \mathrm{mol/L} = 0.025\ \mathrm{M}$

$$[\mathrm{H_3O^+}] = \frac{1.0 \times 10^{-14}}{0.025} = 4.0 \times 10^{-13}\ \mathrm{M} \implies \boxed{\text{basic}}$$

Multi-Step: Neutral pH at body temperature

At 37°C, $K_w = 2.4 \times 10^{-14}$. For neutral water:

$$[\mathrm{H_3O^+}] = \sqrt{2.4 \times 10^{-14}} = 1.55 \times 10^{-7}\ \mathrm{M}$$

$$\mathrm{pH} = -\log(1.55 \times 10^{-7}) = \boxed{6.81}$$

Note: neutral $\neq$ pH 7.00 at temperatures other than 25°C!

4.2 AB7: $K_a = \frac{[\mathbf{H_3O^+}][\mathbf{A^-}]}{[\mathbf{HA}]}$

Simple: K_a from equilibrium data

At equilibrium: $[\mathrm{H_3O^+}] = [\mathrm{A}^-] = 0.0013$ M, $[\mathrm{HA}] = 0.10 - 0.0013 = 0.0987$ M.

$$K_a = \frac{(0.0013)^2}{0.0987} = \boxed{1.7 \times 10^{-5}}$$

Unit Conversion: pH of acetic acid with mass given

$$[\text{HA}] = \frac{3.0/60.05}{0.500} = 0.0999\ \text{M}$$

$$x^2 = 1.8 \times 10^{-5} \times 0.0999 = 1.80 \times 10^{-6} \implies x = 1.34 \times 10^{-3}$$

$$\text{pH} = -\log(1.34 \times 10^{-3}) = \boxed{2.87}$$

Multi-Step: Find K_a and pK_a from pH

Step 1: $[\text{H}_3\text{O}^+] = 10^{-2.72} = 1.91 \times 10^{-3}$ M
Step 2:

$$K_a = \frac{(1.91 \times 10^{-3})^2}{0.20 - 0.00191} = \frac{3.65 \times 10^{-6}}{0.198} = 1.84 \times 10^{-5}$$

Step 3: $pK_a = -\log(1.84 \times 10^{-5}) = \boxed{4.74}$

4.3 AB12: Henderson–Hasselbalch: pH $= pK_a + \log \frac{[\mathbf{A}^-]}{[\mathbf{HA}]}$

Simple: Equal concentrations buffer

$$\text{pH} = 4.74 + \log\left(\frac{0.20}{0.20}\right) = 4.74 + 0 = \boxed{4.74}$$

Unit Conversion: Buffer from masses

$$[\text{HA}] = \frac{6.0/60.05}{0.500} = 0.200\ \text{M}; \quad [\text{A}^-] = \frac{8.2/82.03}{0.500} = 0.200\ \text{M}$$

$$\text{pH} = 4.74 + \log(1.00) = \boxed{4.74}$$

Multi-Step: Buffer after adding strong acid

Step 1: HCl converts A^- to HA:

$$[\text{HA}]_{\text{new}} = 0.50 + 0.010 = 0.51\ \text{M}$$
$$[\text{A}^-]_{\text{new}} = 0.50 - 0.010 = 0.49\ \text{M}$$

Step 2:

$$\text{pH} = 4.74 + \log\left(\frac{0.49}{0.51}\right) = 4.74 + (-0.017) = \boxed{4.72}$$

(Buffer resists pH change: only $\Delta\text{pH} = -0.02$.)

5 Chemical Kinetics

5.1 KN4–5: First-Order Kinetics

Simple: Concentration after time

$$\ln[\mathrm{A}] = \ln(1.0) - 0.050(20) = -1.0 \implies [\mathrm{A}] = e^{-1.0} = \boxed{0.368\ \mathrm{M}}$$

Unit Conversion: Hours to minutes conversion

$t = 6.0$ hr $= 360$ min:

$$\ln[\mathrm{A}] = \ln(0.250) - 0.0025(360) = -1.386 - 0.900 = -2.286$$

$$[\mathrm{A}] = e^{-2.286} = \boxed{0.102\ \mathrm{M}}$$

Multi-Step: Finding k, $[\mathrm{A}]_0$, and time for 95% decomposition

Step 1: $k = \frac{\ln(0.80/0.40)}{40-10} = \frac{0.693}{30} = 0.0231\ \mathrm{min}^{-1}$
Step 2: Back-calculate: $[\mathrm{A}]_0 = 0.80 \cdot e^{0.0231\times 10} = 0.80 \times 1.26 = 1.01$ M
Step 3: 95% gone means $[\mathrm{A}]/[\mathrm{A}]_0 = 0.05$:

$$t = \frac{\ln(1/0.05)}{0.0231} = \frac{\ln 20}{0.0231} = \boxed{130\ \mathrm{min}}$$

5.2 KN5: $t_{1/2} = 0.693/k$

Simple: Half-life from k

$$t_{1/2} = \frac{0.693}{0.0100} = \boxed{69.3\ \mathrm{s}}$$

Unit Conversion: Drug half-life: hours to seconds

$$k = \frac{0.693}{4.0 \times 3600} = \frac{0.693}{14{,}400} = \boxed{4.81 \times 10^{-5}\ \mathrm{s}^{-1}}$$

Multi-Step: Radiocarbon dating

Step 1: $k = 0.693/5730 = 1.21 \times 10^{-4}\ \mathrm{yr}^{-1}$
Step 2: After 20000 yr:

$$\text{fraction} = e^{-1.21\times 10^{-4}\times 20000} = e^{-2.42} = 0.0887 = \boxed{8.87\%}$$

Step 3: For 25% remaining: $0.25 = e^{-kt} \implies t = \ln 4/k = \boxed{11{,}460\ \mathrm{yr}}$ (two half-lives).

5.3 KN8–10: Arrhenius Equation

Simple: Finding k from Arrhenius

$$\begin{aligned} k &= 1.0 \times 10^{10} \times e^{-50000/(8.314\times300)} \\ &= 10^{10} \times e^{-20.04} = 10^{10} \times 1.98 \times 10^{-9} = \boxed{19.8\ \text{s}^{-1}} \end{aligned}$$

Unit Conversion: E_a in kcal to kJ

$E_a = 12.0 \times 4.184 = 50.2$ kJ/mol:

$$\begin{aligned} k &= 5.0 \times 10^{8} \times e^{-50200/(8.314\times400)} \\ &= 5.0 \times 10^{8} \times e^{-15.10} = 5.0 \times 10^{8} \times 2.75 \times 10^{-7} = \boxed{138\ \text{s}^{-1}} \end{aligned}$$

Multi-Step: Two-temperature method for E_a and A

Step 1: Using $\ln \frac{k_1}{k_2} = \frac{E_a}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$:

$$\begin{aligned} \ln \frac{0.010}{0.050} &= \frac{E_a}{8.314}\left(\frac{1}{350} - \frac{1}{300}\right) \\ -1.609 &= \frac{E_a}{8.314}(-4.76 \times 10^{-4}) \\ E_a &= \frac{-1.609 \times 8.314}{-4.76 \times 10^{-4}} = \boxed{28.1\ \text{kJ/mol}} \end{aligned}$$

Step 2: $A = k/e^{-E_a/RT} = 0.010/e^{-28100/(8.314\times300)} = 0.010/1.32 \times 10^{-5} = \boxed{755\ \text{s}^{-1}}$

6 Gas Laws

6.1 GL1: $PV = nRT$

Simple: Volume of 2 mol at STP

$$V = \frac{nRT}{P} = \frac{2.00(0.08206)(273.15)}{1.00} = \boxed{44.8\ \text{L}}$$

Unit Conversion: Pressure of O_2 in atm and mmHg

$n = 3.20/32.00 = 0.100$ mol; $V = 0.500$ L; $T = 310.15$ K:

$$P = \frac{(0.100)(0.08206)(310.15)}{0.500} = 5.09\ \text{atm} = \boxed{3868\ \text{mmHg}}$$

Multi-Step: Gas expansion between tanks

Step 1: $P_1V_1 = P_2V_2$ (isothermal): $P_f = 5.0(10.0)/25.0 = 2.0$ atm.
Step 2: $n = PV/RT = 5.0(10.0)/(0.08206 \times 298.15) = 2.044$ mol.
Step 3: mass $= 2.044 \times 28.02 = \boxed{57.3\text{ g}}$.

6.2 GL8: Graham's Law: $\frac{r_A}{r_B} = \sqrt{M_B/M_A}$

Simple: H_2 vs O_2 effusion

$$\frac{r_{H_2}}{r_{O_2}} = \sqrt{\frac{32}{2}} = \sqrt{16} = \boxed{4.0}$$

H_2 effuses 4 times faster.

Unit Conversion: Unknown gas molar mass

$$\frac{r_{\text{unk}}}{r_{\text{He}}} = 0.355 = \sqrt{\frac{4.00}{M}} \implies M = \frac{4.00}{(0.355)^2} = \boxed{31.7\text{ g/mol}}$$

Multi-Step: Uranium enrichment stages

Step 1: Separation factor per stage:

$$\alpha = \sqrt{\frac{352}{349}} = 1.00430$$

Step 2: Need $\alpha^n = 1.10$:

$$n = \frac{\ln(1.10)}{\ln(1.00430)} = \frac{0.0953}{0.00429} = 22.2 \implies \boxed{23\text{ stages}}$$

6.3 GL11: $u_{\mathbf{rms}} = \sqrt{3RT/M}$

Simple: rms speed of N_2

$$u_{\text{rms}} = \sqrt{\frac{3(8.314)(300)}{0.02802}} = \sqrt{267{,}000} = \boxed{517\text{ m/s}}$$

Unit Conversion: He in km/h

$T = 298.15$ K, $M = 0.00400$ kg/mol:

$$u = \sqrt{\frac{3(8.314)(298.15)}{0.00400}} = 1363\text{ m/s} \times \frac{3600}{1000} = \boxed{4907\text{ km/h}}$$

Multi-Step: Matching rms speeds of two gases

$u_{\text{rms}}(\text{N}_2) = 517$ m/s at 300 K. Set equal for O_2:

$$T_{\text{O}_2} = \frac{u^2 M}{3R} = \frac{(517)^2(0.03200)}{3(8.314)} = \frac{8554}{24.94} = \boxed{343\text{ K}}$$

7 Colligative Properties

7.1 CP5–6: Boiling Point Elevation / Freezing Point Depression

Simple: ΔT_b calculation

$$\Delta T_b = K_b \times m = 0.512 \times 1.50 = \boxed{0.768\,^\circ\text{C}}$$

Unit Conversion: Molar mass from boiling point data

$$\begin{aligned}\Delta T_b &= 100.52 - 100.00 = 0.52\,^\circ\text{C}\\ m &= \Delta T_b/K_b = 0.52/0.512 = 1.016\\ \text{mol} &= 1.016 \times 0.250 = 0.254\\ M &= 15.0/0.254 = \boxed{59.1\text{ g/mol}}\end{aligned}$$

Multi-Step: Antifreeze: ethylene glycol in water

Step 1: mass EG $= 4.00 \times 1000 \times 1.11 = 4440$ g; mol $= 4440/62.07 = 71.5$
Step 2: $m = 71.5/6.00 = 11.9$ mol/kg
Step 3: $\Delta T_f = 1.86 \times 11.9 = 22.1\,^\circ\text{C}$; new FP $= -22.1\,^\circ\text{C}$
Convert depression to °F: $\Delta T_f(^\circ\text{F}) = 22.1 \times 9/5 = \boxed{39.8\,^\circ\text{F depression}}$

7.2 CP7: Osmotic Pressure: $\Pi = MRT$

Simple: Direct calculation

$$\Pi = (0.10)(0.08206)(298) = \boxed{2.45\text{ atm}}$$

Unit Conversion: Concentration from osmotic pressure

$$M = \frac{7.5}{0.08206 \times 298} = 0.307\text{ M} = \boxed{307\text{ mmol/L}}$$

Multi-Step: Molar mass of a protein from Π

Step 1: $\Pi = 3.2/760 = 0.004211$ atm
Step 2: $M = \Pi/RT = 0.004211/24.47 = 1.72 \times 10^{-4}$ mol/L
Step 3: mol $= 1.72 \times 10^{-4} \times 0.100 = 1.72 \times 10^{-5}$
Step 4: Molar mass $= 5.0/1.72 \times 10^{-5} = \boxed{291{,}000 \text{ g/mol}}$

8 Phase Changes

8.1 PC4: Clausius–Clapeyron Two-Point Form

Simple: Finding ΔH_{vap}

$$\begin{aligned}\ln\frac{100}{400} &= \frac{\Delta H}{8.314}\left(\frac{1}{370} - \frac{1}{330}\right)\\ -1.386 &= \frac{\Delta H}{8.314}(-3.27 \times 10^{-4})\\ \Delta H &= \boxed{35.2 \text{ kJ/mol}}\end{aligned}$$

Unit Conversion: Predicting vapor pressure at new temperature

$$\begin{aligned}\ln\frac{44}{P_2} &= \frac{38{,}600}{8.314}\left(\frac{1}{323.15} - \frac{1}{293.15}\right)\\ &= 4643 \times (-3.17 \times 10^{-4}) = -1.472\\ P_2 &= 44 \times e^{1.472} = 44 \times 4.358\\ &= 192 \text{ mmHg} = \boxed{25.6 \text{ kPa}}\end{aligned}$$

Multi-Step: Finding normal boiling point

Step 1: ΔH_{vap}:

$$\begin{aligned}\ln(50/200) &= \frac{\Delta H}{8.314}\left(\frac{1}{340} - \frac{1}{300}\right)\\ \Delta H &= \frac{-1.386 \times 8.314}{-3.92 \times 10^{-4}} = 29.4 \text{ kJ/mol}\end{aligned}$$

Step 2: Normal BP ($P = 760$ torr):

$$\begin{aligned}\ln\frac{50}{760} &= \frac{29{,}400}{8.314}\left(\frac{1}{T_b} - \frac{1}{300}\right)\\ -2.722 &= 3537\left(\frac{1}{T_b} - 0.003333\right)\\ \frac{1}{T_b} &= 0.003333 - 7.70 \times 10^{-4} = 0.002563\\ T_b &= \boxed{390 \text{ K} = 117\,^\circ\text{C}}\end{aligned}$$

9 Atomic Structure & Spectroscopy

9.1 AS1–2: $c = \lambda\nu$ and $E = h\nu = hc/\lambda$

Simple: Frequency from wavelength

$$\nu = \frac{c}{\lambda} = \frac{3.00 \times 10^8}{500 \times 10^{-9}} = \boxed{6.00 \times 10^{14}\ \text{Hz}}$$

Unit Conversion: UV photon energy in eV

$$E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34})(3.00 \times 10^8)}{250 \times 10^{-9}} = 7.95 \times 10^{-19}\ \text{J} = \boxed{4.96\ \text{eV}}$$

Multi-Step: Laser: photons per second

Step 1: $E = hc/\lambda = (6.626 \times 10^{-34})(3 \times 10^8)/(532 \times 10^{-9}) = 3.74 \times 10^{-19}$ J/photon
Step 2: Power = 5.0 mW = 0.005 J/s

$$\text{photons/s} = \frac{0.005}{3.74 \times 10^{-19}} = \boxed{1.34 \times 10^{16}\ \text{photons/s}}$$

9.2 AS3: Rydberg: $1/\lambda = R_\infty(1/n_1^2 - 1/n_2^2)$

Simple: Hα line ($n = 3 \to 2$)

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{9}\right) = 1.097 \times 10^7 (0.1389) = 1.524 \times 10^6\ \text{m}^{-1}$$

$$\lambda = \boxed{656.3\ \text{nm}}$$

Unit Conversion: Lyman limit

$$\frac{1}{\lambda} = R_\infty \left(\frac{1}{1} - 0\right) = 1.097 \times 10^7 \implies \lambda = 91.16\ \text{nm} = \boxed{911.6\ \text{Å}}$$

Multi-Step: Identifying a Balmer transition

$\lambda = 434.0$ nm; $n_1 = 2$:

$$\frac{1}{\lambda} = \frac{1}{434.0 \times 10^{-9}} = 2.304 \times 10^6$$

$$\frac{2.304 \times 10^6}{1.097 \times 10^7} = 0.2100 = \frac{1}{4} - \frac{1}{n_2^2}$$

$$\frac{1}{n_2^2} = 0.2500 - 0.2100 = 0.0400 \implies n_2^2 = 25 \implies \boxed{n_2 = 5}$$

10 Chemical Bonding

10.1 CB1: Bond Order $= \frac{\textbf{bonding } e^- - \textbf{antibonding } e^-}{2}$

Simple: O_2 bond order

$$\text{BO} = \frac{10-6}{2} = \boxed{2.0}$$

Unit Conversion: N_2 vs O_2 comparison

$\text{BO}(N_2) = (10-4)/2 = 3.0 > 2.0 = \text{BO}(O_2)$.
Higher BO $\implies$ shorter bond: 110 pm < 121 pm ✓

Multi-Step: O_2, O_2^-, O_2^{2-} comparison

Species	Bonding e^-	Antibonding e^-	BO
O_2	10	6	2.0 (paramagnetic)
O_2^-	10	7	1.5 (paramagnetic)
O_2^{2-}	10	8	$\boxed{1.0}$ (diamagnetic)

10.2 CB4: $\Delta H = \sum D_{\textbf{broken}} - \sum D_{\textbf{formed}}$

Simple: HCl formation

$$\Delta H = [D(\text{H–H}) + D(\text{Cl–Cl})] - 2D(\text{H–Cl}) = (436 + 242) - 2(431) = \boxed{-184\text{ kJ}}$$

Unit Conversion: Per gram and kcal conversion

$\Delta H = -184$ kJ for 2 mol HCl $= -92$ kJ/mol HCl.

$$\text{Per gram: } -92/36.46 = -2.52\text{ kJ/g}; \quad -92/4.184 = \boxed{-22.0\text{ kcal/mol}}$$

Multi-Step: CH_4 combustion from bond energies

Bonds broken: 4 C–H + 2 O=O $= 4(413) + 2(495) = 2642$ kJ
Bonds formed: 2 C=O + 4 O–H $= 2(799) + 4(463) = 3450$ kJ
$\Delta H = 2642 - 3450 = -808$ kJ
Actual: -890 kJ. Discrepancy ≈ 82 kJ because bond energies are *averages* over many compounds, not exact for any specific molecule.

11 Stoichiometry & Solutions

11.1 ST1: % Composition; ST2: Empirical/Molecular Formula

Simple: % carbon

$$\%\mathrm{C} = \frac{20.0}{50.0} \times 100 = \boxed{40.0\%}$$

Unit Conversion: % oxygen in Al_2O_3

$M = 2(26.98) + 3(16.00) = 101.96$; $\%\mathrm{O} = 48.00/101.96 \times 100 = \boxed{47.1\%}$

Multi-Step: Iron from iron ore

Step 1: %Fe in $Fe_2O_3 = 111.70/159.70 = 69.94\%$
Step 2: mass Fe_2O_3 in 500 kg ore $= 0.724 \times 500 = 362$ kg
Step 3: mass Fe $= 362 \times 0.6994 = \boxed{253\text{ kg}}$

11.2 ST4–5: Molarity and Dilution

Simple: Basic molarity

$$M = \frac{0.50}{2.0} = \boxed{0.25\text{ M}}$$

Unit Conversion: Dilution in mL

Need 2.0 L of 0.10 M from 18.0 M:

$$V_1 = \frac{C_2 V_2}{C_1} = \frac{(0.10)(2000)}{18.0} = \boxed{11.1\text{ mL}}$$

Multi-Step: Serial dilution

1st dilution: $C_2 = 0.200(50.0/500) = 0.0200$ M
2nd dilution: $C_3 = 0.0200(25.0/250) = 0.00200$ M
mol $= 0.00200 \times 0.250 = \boxed{5.0 \times 10^{-4}\text{ mol}}$

11.3 ST9: Percent Yield

Simple: Direct calculation

$$\%\text{yield} = \frac{18.5}{25.0} \times 100 = \boxed{74.0\%}$$

Unit Conversion: Actual yield from % and theoretical

Theoretical $= 2.50 \times 100 = 250$ g; actual $= 0.85 \times 250 = \boxed{212.5\text{ g}}$

Multi-Step: Mg + HCl: theoretical yield and % yield

Step 1: mol Mg $= 10.0/24.31 = 0.411$; 1:1 ratio with H_2
Step 2: theoretical $H_2 = 0.411 \times 2.016 = 0.829$ g
Step 3: $\%\text{yield} = (0.350/0.829) \times 100 = \boxed{42.2\%}$

12 Atomic Mass & Basic Measurements

12.1 BM1: Average Atomic Mass

Simple: Chlorine isotopes

$$0.7577(34.969) + 0.2423(36.966) = 26.50 + 8.96 = \boxed{35.45 \text{ amu}}$$

Unit Conversion: Boron in kg

$\text{avg} = 0.199(10.013) + 0.801(11.009) = 1.993 + 8.818 = 10.81$ amu

$$= 10.81 \times 1.661 \times 10^{-27} = \boxed{1.796 \times 10^{-26} \text{ kg}}$$

Multi-Step: Identifying copper from isotope abundances

$$\begin{aligned} x(62.930) + (1-x)(64.928) &= 63.546 \\ 62.930x + 64.928 - 64.928x &= 63.546 \\ -1.998x &= -1.382 \\ x &= 0.6917 = 69.17\%\ {}^{63}\text{Cu}; \quad 30.83\%\ {}^{65}\text{Cu} \end{aligned}$$

$\boxed{\text{Element: Copper (Cu)}}$

12.2 BM2: Density; BM5: Temperature Conversions

Simple: Density of metal

$$d = 250/28.4 = \boxed{8.80 \text{ g/cm}^3}$$

Unit Conversion: Body temperature to Kelvin

$$T_C = \frac{5}{9}(98.6 - 32) = 37.0\,^\circ\text{C}; \quad T_K = 37.0 + 273.15 = \boxed{310.15 \text{ K}}$$

Multi-Step: Where °C = °F

$$T = \frac{9}{5}T + 32 \implies -\frac{4}{5}T = 32 \implies T = \boxed{-40\,^\circ\mathrm{C} = -40\,^\circ\mathrm{F}}$$

In Kelvin: $-40 + 273.15 = \boxed{233.15\ \mathrm{K}}$

End of Worked Solutions